

ACCEPTANCE SAMPLING WITH ECONOMIC CONSIDERATIONS

A THESIS

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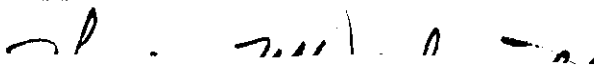
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ACCEPTANCE SAMPLING WITH ECONOMIC CONSIDERATIONS

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CHAPTER I

INTRODUCTION

Under conditions which demand a specific level of quality and which make screening infeasible, sampling plans are necessary. They may be easily determined by use of sampling inspection tables such as MIL-STD-105D. If no specific level of quality has been previously defined, the additional problem of setting some sort of criterion for releasing the product is realized. A most natural criterion from the manufacturer's point of view is to decide on the action (to release or retain the product) which results in the lowest overall cost. The purpose of this thesis will be to develop a mathematical model which may be used to determine a sampling plan which recommends the release or retention of a product depending upon which action results in the lowest cost.

The model will include all costs relevant to sampling along with the probabilities the costs will actually occur. The cost function will then be optimized for a given underlying distribution of the number of defectives to find a sampling plan which results in minimum total production cost.

It will be shown that a model based on economics is most appropriate for it can be easily modified to contain

additional criteria by the introduction of artificial costs and constraints.

Description of the Problem

The quality control system is in a unique position within the framework of the manufacturing process. It is supported by the producer who wishes to minimize the cost of production but its purpose is to insure high quality in the final product which can lead to higher production costs. These two objectives are only compatible if the quality control system is viewed as a means of establishing the total cost of manufacturing--including the cost of bad quality--and optimizing with all factors considered.

The problem of establishing the total cost of manufacturing is not simple, since all relevant costs must be determined and the conditions under which each of these costs is actually incurred must be known. This information should then be used to weigh each component of the cost model by the probability of occurrence.

Assuming that inspection is necessary, a single-sampling attribute inspection plan will be used. That is, in a sample of size n , if $c+1$ or more defectives are found, the entire population from which the sample was taken is rejected. Otherwise it is accepted. Therefore in the cost model to be optimized, the unknowns will be the sample size (n) and the acceptance number (c).

Purpose of the Research

The purpose of this study is to formulate a model for determining the appropriate level of inspection sampling for any manufacturing process. Since the manufacturer is naturally concerned with profits and therefore minimizing the cost of production, the model will be designed with the objective of minimizing total cost.

The secondary objective is to design a model having general applicability in a variety of production operations.

Method of Procedure

Chapter II of this thesis is an outline of the published literature which is relevant to the problem of determining a sampling plan resulting in minimum total production cost. Included in this literature are models and solution procedures based on Bayesian techniques and having applications in determining single-sampling attribute inspection plans. In order to establish a new approach, an understanding of the problem and previous attempts at its solution is essential.

The purpose of the next chapter is to introduce the cost parameters applicable to production, inspection, and unsatisfactory products. From these cost components a model is formulated. Under certain assumptions, the expected cost will be determined as a function of sample size and acceptance number. A discussion of the assumptions as well as the

appropriateness of the model is included.

The fourth chapter is devoted to minimization of the expected cost. An iterative solution procedure is presented. A computer program is used to find the sampling plan which is most desirable. There is a discussion of the program and an analysis of the sensitivity of the model.

The final chapter is a summary of the analysis. The conclusions from the present research and the recommendations for further research areas are reviewed also.

CHAPTER II

BACKGROUND

Introduction

Quality control can be a perplexing problem. At what level of quality should the producer be satisfied? When producing life-preservers one cannot afford defectives. But, if manufacturing salad forks, more deviation is acceptable in the process.

Looking at the problem from the producer's short range view, the quality level to be maintained is that which costs the least to manufacture. However, in the long run, the producer wishes to operate at that level of quality which minimizes all costs (not only the manufacturing costs).

The cost categories which will be assumed to be related to outgoing quality level costs are production costs, acceptance costs, and unsatisfactory product costs. The producer must weigh these costs against each other and thereby determine the level of inspection which is necessary to minimize the total cost of the product for a specific level of outgoing quality.

Previous Approaches to the Problem

There has been much interest in determining an

"economic" based single stage acceptance sampling plan. But few authors have approached the problem by first determining all the costs which add to quality expenditures along with the probabilities that the costs occur and then minimizing these costs.

A Summary of Published Models

An interesting model used to determine the lot percent defective for which it is economically feasible to begin sampling was proposed by Cyrus Martin [11]. By comparing the costs incurred under a sampling plan and the costs incurred with no inspection, the point to begin sampling (the breakeven point) is found. If the process fraction of defective items is as high as this breakeven point, inspection is required. Otherwise sampling is not economically justifiable.

Martin's "Combined Cost" for inspection per lot expression includes the cost of inspecting the lot, replacing units, and when a defective is not discovered, accepting the defective. The "no-inspection cost" per lot expression is only the cost of accepting the defectives within the lot. It is assumed that the fraction of defective items is known and that a sampling plan has been predetermined.

Martin's model is especially interesting because it is a plan to determine when a specific inspection procedure should be used according to economics. The model proposed

in this thesis could be considered a generalization of Martin's ideas since this new model is designed to determine exactly the sampling plan which is most economically feasible.

Other researchers have primarily approached the problem from two viewpoints. One has been an attempt to minimize the losses associated with accepting a batch or perhaps rejecting a batch or even both. The other approach has been to estimate the number of defective items in the lot by using the sampling plan which minimizes the posterior variance of the number of defectives subject to a cost of sampling constraint.

A. Hald [9] has developed a model which includes the cost of inspection, the expected loss due to accepted defectives, and the expected loss from rejected lots. The total cost function is not in terms of cost but is in terms of the lot fraction defective.

$$K(n,c) = nk_s + \sum_{x=0}^c g_n(x)E(X-x|x) \\ + (N-n)k_r \sum_{x=c+1}^n g_n(x)$$

where

N = lot size

n = sample size

X = number of defectives in lot

x = number of defectives in sample

c = acceptance number

$g_n(x)$ = marginal distribution of x , or average probability of the number of defectives in sample

$E(X-x|x)$ = expected number of defectives in the non-inspected part of the lot when x defectives have been found in the sample

k_s = costs of sampling inspection (per unit)/ costs of accepting a defective item

k_r = costs of sorting inspection (per unit)/ cost of accepting a defective item.

Hald has derived two inequalities which determine the optimal solution to $K(n, c)$. Solution of these inequalities leads to an interval for n associated with a given c .

In a later paper Hald [8] suggests solving the problem by approximating the optimal solution. He proves, for the optimal plan, the average decision loss is asymptotically equal to the sampling costs, but formulation of even an approximate solution from the inequalities is difficult.

The model which is proposed by D. Guthrie and M. V. Johns [6] is similar to that of Hald. They assume that the lot of N items can be characterized by N non-negative random variables X_i ($i = 1, \dots, N$). For example, if the i th item is defective $x_i = 1$, otherwise $x_i = 0$.

Let n = sample size

$$S_k = \sum_{i=0}^k x_i$$

$a_1x_i + a_2$ = cost of accepting the i th item; a_1 is a cost depending on the i th item and a_2 is a fixed cost;

$r_1x_i + r_2$ = cost of rejecting the i th item; r_1 is a cost depending on the i th item and r_2 is a fixed cost;

$s_1x_i + s_2$ = cost of inspecting the i th item; s_1 is a cost depending on the i th item and s_2 is a fixed cost.

The cost of acceptance is the cost of accepting the uninspected items plus the cost of inspecting the sample, or

$$a_1(S_N - S_n) + a_2(N - n) + s_1S_n + s_2n.$$

The cost of rejection is the cost of rejecting the uninspected items plus the cost of inspecting the sample, or

$$r_1(S_N - S_n) + r_2(N - n) + s_1S_n + s_2n.$$

To find the optimal sample size, Gutherie and Johns have derived "explicit asymptotic characterizations for large N of the decision procedures and sample sizes which are optimal in the Bayes sense for various classes of a priori

probability distributions" [6]. These characterizations are not useful in practical problems because they are too cumbersome, but they do lead to the optimal solution for the above model.

G. Horsnell [10] has approached the problem with a slightly different point of view. He has assumed that a manufacturer can do little to change his production process quality level after the process is established. Under this assumption, a consumer can expect a given level of quality. Horsnell, then, has based his cost model purely on the cost per effective item and did not allow for inclusion of a penalty for bad material which is sent out to a consumer and later rejected.

Assuming non-destructive sampling, Horsnell has calculated the average cost of sampling the lot as follows.

$$\bar{C} = NC_m + C(n_s, n_i)$$

where

N = lot size

n_s = number of items sampled

n_i = number if items inspected

C_m = average cost of manufacturing per unit

$C(n_s, n_i)$ = average cost per lot due to the labor of applying the sampling scheme.

The "cost of manufacturing" must tacitly include the cost of

screening and reworking material when necessary.

Horsnell calculates the "effective cost" per accepted item as the average cost per sampled lot divided by the fraction of lots of normal production which will be accepted. That is, the cost per accepted item is

$$C = \frac{NC_m + C(n_s, n_i)}{N \int_0^1 A(p)y(p)dp}$$

where

p = fraction of defective items

$A(p)$ = fraction of lots containing 100p percent defectives

$y(p)$ = process curve (of normal production).

He has reasoned that the probability of accepting a lot with 100p percent defectives under a sampling plan with parameters $(n_1, k+1)$ is greater than the probability of accepting the same lot under a plan of (n_0, k) and that is greater than the probability of accepting the lot under the plan of $(n_{-1}, k-1)$ where $n_1 > n_0 > n_{-1}$. Finally, Horsnell has concluded that the cost of accepting an item will be a global minimum if

$$C_{n_{-1}} \geq C_{n_0} \leq C_{n_1} \text{ for } n_{-1} < n_0 < n_1.$$

For a given process curve and lot size, tables can be prepared which will show the most economical sampling plan for a given value of the ratio $C_n/(\text{cost per unit sampled})$, and having a specified consumer's risk, and accepting as much of the product as possible.

A modified view of Horsnel's ideas has been taken by G. W. Churchill [1]. Churchill has allowed for a decision to scrap, sort, or pass the lot. He has devised a cost model to determine the sample size and the first and second acceptance numbers. If the number of defectives is greater than the second acceptance number but less than or equal to the first acceptance number, the lot will be sorted but not scrapped. If the number of defectives is less than the second acceptance number the lot will be passed. Otherwise it will be scrapped.

To minimize the ratio of the total cost, C_t , to the number of good units, G_u , the following cost equation is optimized.

$$\frac{C_t}{G_u} = \frac{Nc_p + nC_s \sum (1-p_i)p_{1i}x_i + NC_a \sum p_i x_i + NC_a \sum (1-q_i)(1-p_i)p_{1i}x_i}{N \sum (1-q_i)p_{1i}x_i}$$

where

N = lot size

n = sample size

- c_1 = first acceptance number
 c = second acceptance number
 p_1 = probability of acceptance associated with n , c_1 and fraction defective
 p = probability of acceptance associated with n , c and fraction defective
 q = fraction defective
 q_a = average fraction defective
 q_i = fraction defective in i th fraction of lots
 x_i = fraction of lots that are q_i fraction defective
 p_{1i} = probability of acceptance associated with n , c_1 and q_i
 p_i = probability of acceptance associated with n , c , and q_a
 C_s = cost of inspecting one unit
 C_p = cost of units prior to inspection point
 C_a = cost of all operations after inspection point and prior to the operation which will remove the defects.

The solution method which is recommended is that of trial and error. The sample parameters should be varied to attain the minimum value of C_t/G_u . A computer program is included to find the acceptance number and cost per effective unit for a given sample size.

G. B. Witherill [14] has proposed a model for establishing a "neutral line" which is a "locus of points (n, c_n) such that...the loss associated with accepting a batch is equal to that of rejecting it." The neutral line consists of points where the average overall quality levels

of the cost of acceptance at each quality level weighed with the probability of that quality level occurring is equal to the average overall quality levels of the cost of rejecting at each quality level, weighed by the probability of that level occurring.

The solution method of this model is that of successive iterations. A value of n is improved by guessing its true value and correcting the value to fit an explicit expression. Convergence is not demonstrated although it appears that it occurs after a few iterations.

The problem that is considered by S. Zacks [15] is to determine the optimal sample size within a specified cost level. The definition of this optimal sample size is as follows. A sample of size n which minimizes the variance of the estimator of defective items per lot will be called optimal. He did not consider the minimization of costs as the criterion for an optimal sample size.

Zacks has derived a relationship for the sample size, the lot size, and a given prior distribution of the percent defective (subject to a budgetary constraint). From this relationship, the optimal sample size can be calculated. Unfortunately, Zacks did not find a total sampling plan (i.e. sample size and acceptance number), but his plan is noteworthy since W. A. Ericson [3] has proven this sample size is also optimal in the sense of minimization of the posterior expected loss (under certain conditions). Ericson's model follows.

Let $\mu = (\mu_1, \dots, \mu_k)$ be a vector of unknown means of k independent normal processes with known variances σ_i^2 ($i = 1, \dots, k$). Suppose that μ has a k -dimensional normal prior distribution with mean m' (a vector) and positive definite covariance matrix $V' = (N')^{-1}$. Independent samples of size n_i are drawn from the process. The vector $\bar{X} = (\bar{X}_1, \dots, \bar{X}_k)$ is the observed means of these samples. The posterior distribution of μ given \bar{X} is k -dimensional with mean $m'' = (m'N' + \bar{X}N)(N + N')^{-1}$ and covariance matrix $V'' = (N + N')^{-1}$ where N is a diagonal matrix with the elements n_i/σ_i^2 on the diagonal. Let a be an action whose utility depends on the unknown parameter μ . Then the loss resulting from the sequence of choosing $n = (n_1, \dots, n_k)$ which observes \bar{x} and choosing a when μ is true is

$$\ell(n, \bar{x}, a, \mu) = \ell(a, \mu) + cn^t$$

where $\ell(a, \mu) \geq 0$

$c \geq 0$ is the cost per unit of the observation.

Ericson notes that the posterior expected value of the loss is a function of m'' and $v''(n)$. He then proves that, under the above assumptions, the posterior expected loss will be minimized by choosing n such that the posterior variance (which is a function on n) subject to the constraint $cn^t = c_0$ where c_0 is that value of $c \geq 0$ which minimizes

$$f_{\ell}(v^*(c)) + cn^t$$

for any given $f_{\ell} \geq 0$ and

$$f_{\ell}(v''(n)) = \frac{E}{\bar{x}|n} \min_{\mu|\bar{x},n} E'' \ell(a, \mu).$$

The solution to this problem is given by Ericson [4] in an earlier paper.

The problem of determining an economic sampling plan for a multistage manufacturing process has been considered by E. W. Stacy, L. E. Hunsinger, and J. F. Price [13]. Their model is of interest here since it can be modified to fit a single stage manufacturing process. The expected cost of rejected lots is

$$nT + (N-n)s + a(\alpha t + \beta u),$$

and the expected cost of accepted lots is

$$E(\text{cost}) = \frac{\sum_{r=0}^{d_0} \{ (r(\alpha t + \beta u) + (a-r) \sum_{s=2}^{d_0} (\alpha_s t_s + \beta_s u_s) \lambda_s v_s P(R=r|D=0) \}}{\sum_{r=0}^{d_0} P(R=r/D=a)},$$

where

N = lot size

n = sample size

d = acceptance number

λ_s = probability that if an item fails to pass inspection at the first stage, the defective will be troublesome at the s th stage

v_s = probability that an item will be troublesome at the s th stage, given that inspection reveals a defect that may or may not cause quality degradation at a subsequent stage

$p_s = \lambda_s v_s$ = probability that an item that would be judged defective at the first stage will be troublesome at the s th stage

$q_s = 1 - p_s$

D = the number of defective items in a lot; a random variable

a = the number of defective items in a lot

R = the number of defective items in a sample; a random variable

r = the number of defective items in a sample

α = probability that a defective found at the inspection station can be reworked

$\beta = 1 - \alpha$

t = cost of reworking a defective item

u = cost of replacing a defective item.

An experimental computer program is available through IBM to determine the optimal solution to the above cost equation.

An approach similar to the approach that is suggested in this thesis is that of D. E. Morgan [12]. He has developed the "Quico System" or Quality Improvement through Cost Optimization. He considers five cost components: quality

creation, quality and defect influence, internal resultant, external resultant, and invariant costs. Through the 70 responses to a questionnaire that has been sent to almost 200 companies, he has determined that the data necessary to assess these costs is available within most companies supporting a quality control program.

Morgan did not consider such costs as customer dissatisfaction and did not allow for the possibility that certain costs may or may not occur under different predictable or unpredictable situations. Instead, he has relied heavily on the assumption that a company can accurately predict all costs for each time period.

R. B. Fetter [5] has designed a highly specific cost model appropriate for a tooling process. Included in the cost components is the inspection cost, the cost of defective items, and the checking cost or the cost of checking the setup before production begins to be sure the machinery is in order. He has assumed the Poisson distribution for the number of defective items in a lot and calculates the expected cost as

$$E(\text{cost}) = c_i n + c_k (1 + \sum_j P_j P_{aj}) + c_o [(s-n) (\sum_j P_j P_{aj} p_j + \sum_j (1-P_{aj}) P_j)],$$

where

c_i = cost of inspection per unit

c_k = cost of checking setup

- c_d = cost of defectives per unit
- P_j = probability of j th state
- P_{aj} = probability of acceptance in j th state
- s = run (lot) size
- n = sample size
- p_j = fraction defective in j th state.

Fetter has provided a computer program for solution of this model and notes that other models that are based on the same ideas can be created when they are needed.

Although the model is specific the cost components are generalized. The cost of defectives is not easily calculated and requires some detailing.

Conclusions

The models that are presented in this chapter lead to developing a model that is based on the costs for producing effective items and defective items. By associating the probabilities of producing either an effective or defective with its cost, the overall average cost can be determined and an attempt can be made to minimize that cost. One of the objectives to be accomplished in this thesis is to develop such a model.

CHAPTER III

MODEL DEVELOPMENT

The model that is developed in this research is a representation of the manufacturing process. The modeling system consists of the multiple cost components that are developed with the objective of determining a sampling design in the interest of minimizing the cost of the manufacturing process.

General Model

A general formulation of the model of the manufacturing process is

minimize: Total System Costs.

subject to: 1. Meeting all company quality policies.
2. Meeting all customer quality demands.
3. Staying within all cost bounds.

Introduction of Cost Parameters

The problem is to estimate the costs of good quality, bad quality, and inspection, and then to determine a balance among the three. The cost components must not only be defined, but also weighed by the probability that the costs actually occur.

The costs fall into three categories: production

costs, unsatisfactory product costs, and acceptance costs.

Embedded in the representations of the cost components is the idea that a bar above the representation indicates a constant (e.g. \bar{C}_m) and no bar indicates a function (e.g. C_s).

Production Costs

1. The production costs obviously include the cost of materials and labor. \bar{C}_m will be defined as the (average) cost per unit produced due to materials and labor. It is assumed through this definition that \bar{C}_m is a constant.

2. Screening of rejected lots saves those items in rejected lots which are within the design specifications. Each item within the lot will be categorized as an effective, an item to be reworked, or a defective. C_s will be defined as the cost per lot screened and will be applied only to the lots requiring screening. It will be considered as a function having parameters which must be determined for each product and process. It is possible that C_s includes a relationship to the number of defective items in the lot and the extent of the defect. The average cost of screening could be approximated from past testing.

3. Spoilage costs result from products which are discarded for not meeting the design specifications. \bar{C}_D , the spoilage costs per unit rejected, will be defined as the difference in revenue between an effective item and a defective item. If there is some scrap value associated with a defective, it will be accounted for within this definition

of spoilage costs. It is assumed through this definition that \bar{C}_D is a constant.

4. Rework costs are those additional costs necessary to make a product acceptable. C_R , the cost per unit made over will be defined as the additional cost necessary to correct a defective item. C_R is considered as a function having parameters which must be determined for each product and process. It can include a relationship to the extent of the defect or to other factors. The reworked items are classified as effectives after being adjusted and will be sold as such.

Unsatisfactory-Product Costs

Unsatisfactory-product costs are those costs resulting from acceptance of products outside the design specifications. They primarily consist of the costs due to the loss of customer good will and the cost of guarantee and warranty work.

The Average Outgoing Quality of a lot or AOQ is approximated by the fraction of defective items times the probability of accepting a lot given it has that fraction of defective items:

$$AOQ = (d/N)P(\text{Acceptance} | (d/N)).$$

An appropriate form of the unsatisfactory-product costs is an equation which is zero when the Average Outgoing Quality is at an acceptable level and has a positive cost

when the Average Outgoing Quality is greater than that level. This acceptable quality level will be called AOQ^* and must be evaluated for each product as its minimum acceptable quality level. The unsatisfactory product cost will be positive when the value of $AOQ > AOQ^*$. When the unsatisfactory-product cost equation has a positive value, the per unit defective cost will be

$$\bar{U}(AOQ - AOQ^*) \quad \text{if under warranty or guarantee}$$

and

$$(U + aC_D + (1-a)C_R)(AOQ - AOQ^*) \quad \text{if under warranty or guarantee}$$

and where a = fraction of time
a defective must be replaced
rather than reworked.

For purposes of discussion in this thesis, the case of warranty will be considered appropriate.

The unsatisfactory product cost is defined as the cost of loss of customer good will due to lower quality than agreed upon plus the cost of correcting the defectives (a per unit defective cost). \bar{U} is defined as the average cost of loss of customer good will and is a constant. To further describe \bar{U} , a value can be placed on each sales account and the total value of lost accounts in a period divided by the estimated total number of defectives produced in that period

to get a per defective cost. An alternative description of \bar{U} is to let \bar{U} be the loss in sales revenue in a period divided by the estimated total number of defectives produced in that period.

\bar{C}_D is the spoilage cost per unit, C_R is the cost per unit reworked. Both costs have been defined previously.

Acceptance Costs

Acceptance costs are those resulting from expenditures due to inspection. Included are the costs of a quality control team, instruments, and in the case of destructive sampling, loss of revenue from the products sampled (not the cost of the product). This definition is necessary in order that acceptance costs be analogous to production and unsatisfactory products costs. An appropriate form of the acceptance cost equation is $\bar{C}_A + C_A$ (number of items inspected). \bar{C}_A is a "set up" cost for the inspection system and C_A is a per item sampled expenditure. C_A is a function whose parameters must be evaluated for each product and process. C_A may rise exponentially with the number of items tested or have a relationship to other factors.

Assumptions

1. All items tested are correctly identified and treated as either effective items, defective items or items to be reworked.

2. The lot size is known.

3. The distribution of the number of defectives in a lot is known. This distribution may be determined by inspecting a large number of lots and recording how often a number of defects, d for $d = 0, \dots, N$, occurs in a lot.

Definitions

N = number of items in a lot

n = number of items in a sample

$N' = N - n$

c = acceptance number

D = number of defective items in a lot; a random variable which may take on any of the values

$d = 0, \dots, N$

R = number of defective items in a sample; a random variable which may take on any of the values

$r = 0, 1, \dots, \min(n, d)$

$\delta(d) = P(d \text{ defects in a lot})$

a = fraction of defective items replaced

$1 - a$ = fraction of defective items reworked

$C(d, r, n, c)$ = cost of a lot with d defective items and with r defective items found under a sampling plan of parameters (n, c)

$K(n, r) = \binom{n}{r}$

$x = \min(c, d)$

$y = \min(n, d)$

Introduction of Probabilities

To find the conditional probability that $R = r$ given that $D = d$ recall the [7]

hypergeometric probability function provides probabilities of certain events when a sample of n objects is drawn at random from a finite population of N objects where the sampling is done without replacement and where each element of the population may be dichotomized in some simple fashion as belonging to one of two disjoint classes.

Then by definition

$$P(R=r|D=d) = \begin{cases} \binom{d}{r} \binom{N-d}{n-r} / \binom{N}{n} & \text{for } r=0, \dots, \min(d, n) \\ 0 & \text{elsewhere.} \end{cases}$$

Recall

$$P(x_2|x_1) = P(x_1, x_2) / P(x_1)$$

or

$$P(x_1, x_2) = P(x_2|x_1)P(x_1)$$

and

$$P(x_2) = \sum_{x_1} P(x_1, x_2) = \sum_{x_1} P(x_2|x_1)P(x_1).$$

Then

$$P(D=d, R=r) = P(R=r|D=d)P(D=d)$$

and

$$\begin{aligned}
 P(\text{r defectives in a sample}) &= P_r = P(R=r) \\
 &= \sum_{d=0}^N P(R=r | D=d) \delta(d) \\
 &= \sum_{d=0}^{N'} (K(N-d, n-r) K(d, r) / K(N, n)).
 \end{aligned}$$

$$\begin{aligned}
 P(\text{accepting a lot}) &= \sum_{r=0}^C P(R=r) \\
 &= \sum_{r=0}^x \sum_{d=0}^{N'} (K(N-d, n-r) K(d, r) / K(N, n)) \delta(d).
 \end{aligned}$$

$$\begin{aligned}
 P(\text{Accepting a lot} | D=d) &= \sum_{r=0}^C P(R=r | D=d) \quad (1) \\
 &= \sum_{r=0}^x K(N-d, n-r) K(d, r) / K(N, n).
 \end{aligned}$$

$$AOQ = (d/N) P(\text{accepting a lot} | D=d) \quad (2)$$

$$= (d/N) \sum_{r=0}^x K(N-d, n-r) K(d, r) / K(N, n).$$

$$P(\text{screening a lot}) = P_s = P(\text{rejecting a lot}) \quad (3)$$

$$= 1 - P(\text{accepting a lot})$$

$$= 1 - \sum_{r=0}^x \sum_{d=0}^{N'} (K(N-d, n-r) K(d, r) / K(N, n)) \delta(d).$$

For one illustration of the cost model see Figure 1. The following expressions are the total cost expressions (and the associated probabilities) for a lot which is accepted by both the producer and consumer, accepted by the producer but rejected by the consumer, and rejected by the producer.

1. $(N\bar{C}_m + nC_A)(1-P_s)P(AOQ \leq AOQ^*)$
2. $(N\bar{C}_m + nC_A + (\bar{U} + a\bar{C}_D + (1-a)C_R)(AOQ - AOQ^*)N\theta)$
 $\times (1-P_s)P(AOQ \leq AOQ^*)$
3. $(N\bar{C}_m + nC_A + C_s)(1 - \frac{d-r}{N-n})P_s$
 $+ (N\bar{C}_m + nC_A + \bar{C}_s + C_D a(N-n) + C_R(1-a)(N-n)) \times (\frac{d-r}{N-n})P_s$

The expression for Figure 1 follows. For further explanation see equations 1, 2, and 3.

$$\begin{aligned}
 C(d, r, n, c) = & \bar{C}_m, N \\
 & + C_s P_s \\
 & + \bar{C}_D (P_r \cdot r + P_s \delta(d)(d-r))a \\
 & + C_R (P_r \cdot r + P_s \delta(d)(d-r))(1-a) \\
 & + (\bar{U} + a\bar{C}_D + (1-a)C_R)(AOQ - AOQ^*)d \\
 & + \bar{C}_A + C_A n
 \end{aligned}$$

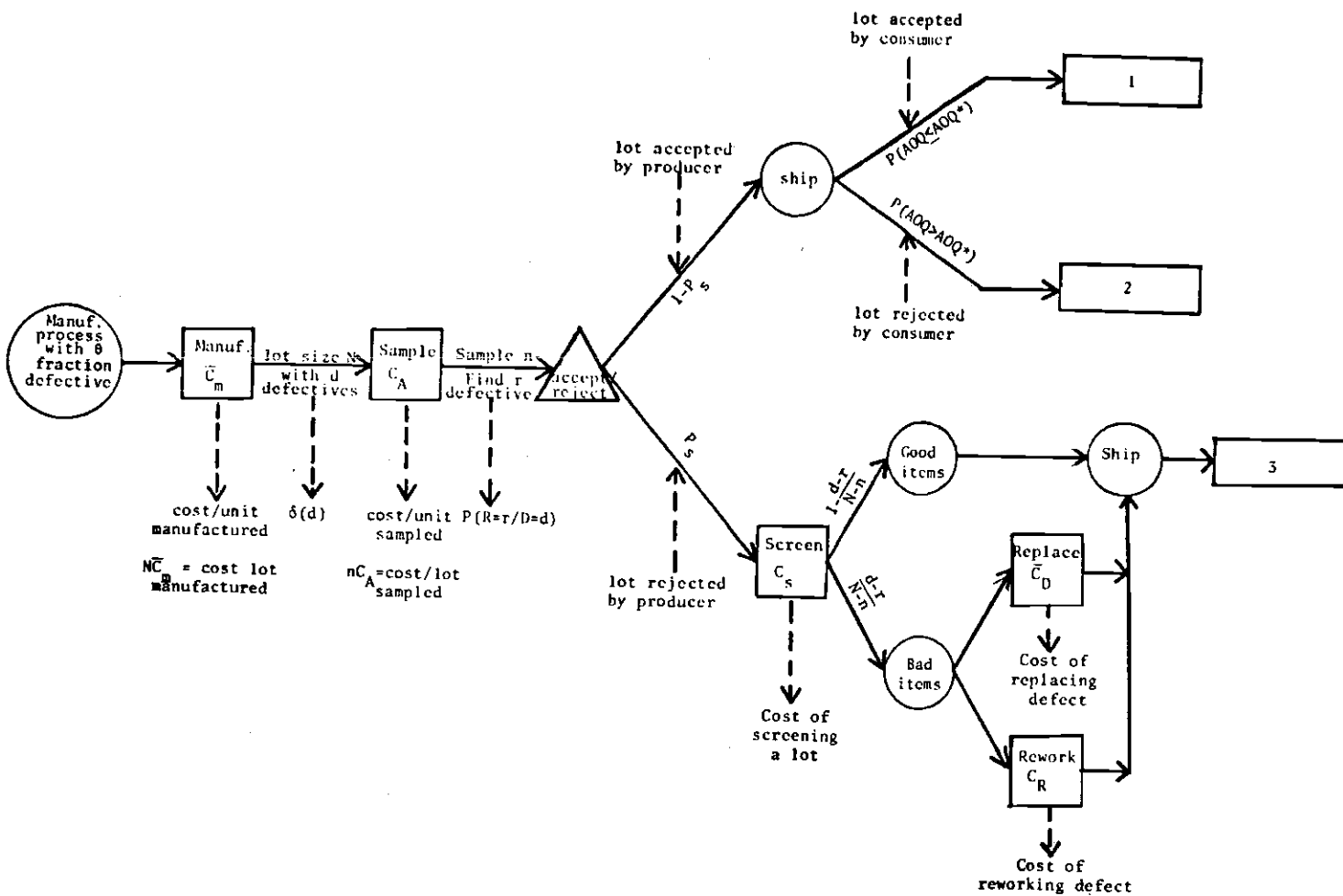


Figure 1. Illustration of the Cost Model

If C_s , the screening cost per lot, and C_R , the reworking cost per unit are not functions of the number of defective items, then the expected value of the cost of a lot taken over the number of defective items can be calculated for general costs. To assume C_s and C_R are not function of d may or may not be a reasonable assumption for a specific process. Expressions for C_s and C_R must be developed for the process in mind. It must then be determined if this assumption is valid. If not, the expected costs due to screening and reworking must be calculated and then the minimum cost must be found in a method similar to that proposed in this research.

In Chapter IV it is the objective of the author to examine the expected cost equation under the assumption that the screening and reworking cost functions are not functions of the number of defective items.

Under that assumption

$$E_{d,r}(C(d,r,n,c)) = \bar{C}_m N$$

$$+ C_s P_s$$

$$\begin{aligned}
& + \bar{C}_D a (E(R) + P_S E(D-R)) \\
& + C_R (1-a) (E(R) + P_S E(D-R)) \\
& + (\bar{U} + a\bar{C}_D + (1-a)C_R) E(d(AOQ-AOQ^*)) \\
& + \bar{C}_A + C_A n \\
= & \bar{C}_m N \\
& + C_S [1 - \sum_{d=0}^{N'} \sum_{r=0}^x (K(N-d, n-r) K(d, r) / K(N, n)) \delta(d)] \\
& + \bar{C}_D a [\sum_{d=0}^{N'} \sum_{r=0}^y r (K(N-d, n-r) K(d, r) / K(N, n)) \delta(d) \\
& + [1 - \sum_{d=0}^{N'} \sum_{r=0}^x (K(N-d, n-r) K(d, r) / K(N, n)) \delta(d)]] \\
& \times \sum_{d=0}^{N'} \sum_{r=0}^y (d-r) (K(N-d, n-r) K(d, r) / K(N, n)) \delta(d)] \\
& + C_R (1-a) [\sum_{d=0}^{N'} \sum_{r=0}^y r (K(N-d, n-r) K(d, r) / K(N, n)) \delta(d) \\
& + [1 - \sum_{d=0}^{N'} \sum_{r=0}^x (K(N-d, n-r) K(d, r) / K(N, n)) \delta(d)]] \\
& \times \sum_{d=0}^{N'} \sum_{r=0}^y (d-r) (K(N-d, n-r) K(d, r) / K(N, n)) \delta(d)]
\end{aligned}$$

$$\begin{aligned}
& + (\bar{U} + a\bar{C}_D + (1-a)C_R) \left[\sum_{d=0}^{N'} (d\delta(d)/N) \sum_{r=0}^x K(N-d, n-r) K(d, r) K(N, n) \right. \\
& \quad \left. - AOQ^* \right] \sum_{d=0}^N d\delta(d).
\end{aligned}$$

A Discussion of the Assumptions

The assumption that any item inspected will be correctly identified can be relaxed to allow for inspection errors if they are expected. A paper by R. D. Collins, et al. [2] indicates that the true fraction defective, p , should be modified to the "apparent" fraction defective, p_e , if inspection errors are expected. Letting e_1 be the probability that an effective is classified as a defective and e_2 be the probability that a defective is classified as an effective then

$$p_e = p(1-e_2) + (1-p)e_1$$

The process's percent defective distribution can be modified by substituting p_e for $E_d(\delta(d))$ in the cost model.

The other two assumptions that the lot size and the underlying distribution of the number of defectives is known are quite natural and need no discussion.

Appropriateness of the Model

A producer knows that his product must please his

consumer to prevent him from taking his business elsewhere. It would be unreasonable for him to put a product on the market without faith that the product will perform as claimed and it should be just as unreasonable for the producer to put each unit of the product on the market without faith that the units will perform as expected. To establish this faith some sort of testing procedure is necessary. To decide on the number of items to be tested, some knowledge of the cost of inspection must be obtained, for at some point, knowledge of the product gained by inspection would not be worth the cost.

The cost model developed in this thesis has been designed to aid in deciding the appropriate level of inspection. A consumer's requirements for a specific level of quality or AOQ may be used as a basis to restrict the set of feasible sampling plans. To do this an appropriate constraint concerning the customer's demands can easily be added to the model. If a customer requires a certain level of inspection, a constraint assuring this level should be added to the model. It should not be assumed that the required amount of inspection determines the sampling plan. Other costs may outweigh the sampling costs. In that case, the optimal solution may require a more extensive plan than specified.

Just as consumer requirements may restrict the set of feasible sampling plans, so may company policy. A high artificial cost may be added to the model to edge away from

sampling plans which do not conform to the company's policy. If there is an upper bound on any type of cost, a constraint can be added to the model forcing the cost below that bound.

By adding constraints and artificial costs to the model, the model can be adapted to fit specific situations. The ability to fit a variety of cases fulfills a requirement of the model proposed in the first chapter, thus the model is appropriate for the stated specifications.

CHAPTER IV

SOLUTION OF THE MODEL

In this chapter the expected cost equation will be examined in order to suggest a method to minimize the expected cost with respect to the number of items to be sampled, n , and the acceptance number, c . The selected sampling plan will be those values of n and c for which this expected minimum cost is minimized.

In Chapter III it was assumed that the distribution of the number of defective items is known. In this chapter it will be further assumed that the number of defective items has the binomial distribution.

The binomial distribution will be assumed to be the underlying distribution of the number of defectives in a lot. In n independent trials with probability θ of an event occurring (finding a defective), the probability of d events occurring (finding d defectives) is

$$\delta(d) = K(N,d)\theta^d(1-\theta)^{N-d}.$$

θ is the overall process fraction defective and has been evaluated from knowledge gained in previous testing.

The binomial distribution has been substituted into

equation (4) for the distribution of the number of defec-
tives. The new cost equation is

$$\begin{aligned}
 E_{d,r}(C(d,r,n,c)) &= \bar{C}_m \cdot N \\
 &+ C_s \left[1 - \sum_{d=0}^{N'} \sum_{r=0}^x K(N-n, d-r) K(n, r) \theta^d (1-\theta)^{N-d} \right] \\
 &+ a \bar{C}_D \left[\sum_{d=0}^{N'} \sum_{r=0}^y r K(N-n, d-r) K(n, r) \theta^d (1-\theta)^{N-d} \right. \\
 &\quad + \left(1 - \sum_{d=0}^{N'} \sum_{r=0}^x K(N-n, d-r) K(n, r) \theta^d (1-\theta)^{N-d} \right) \\
 &\quad \times \left. \sum_{d=0}^{N'} \sum_{r=0}^y (d-r) K(N-n, d-r) K(n, r) \theta^d (1-\theta)^{N-d} \right] \\
 &+ (1-a) C_R \left[\sum_{d=0}^{N'} \sum_{r=0}^y r K(N-n, d-r) K(n, r) \theta^d (1-\theta)^{N-d} \right. \\
 &\quad + \left(1 - \sum_{d=0}^{N'} \sum_{r=0}^x K(N-n, d-r) K(n, r) \theta^d (1-\theta)^{N-d} \right) \\
 &\quad \times \left. \sum_{d=0}^{N'} \sum_{r=0}^y (d-r) K(N-n, d-r) K(n, r) \theta^d (1-\theta)^{N-d} \right] \\
 &+ (\bar{U} + a \bar{C}_D + (1-a) C_R) [(1/N) \\
 &\quad \times \sum_{d=0}^{N'} \sum_{r=0}^x K(N-n, d-r) K(n, r) \theta^d (1-\theta)^{N-d} - \text{AOQ*}]
 \end{aligned}$$

Call the probability associated with C_S , P'_S . Call the probability associated with $a\bar{C}_D$, P'_D . P'_D is also the probability associated with $(1-a)C_R$. Call the weight value associated with the occurrence of the unsatisfactory product costs, P_U .

Computer Solution

Close examination of the above expression reveals that the more cumbersome calculations can be easily carried out on a computer. The unknowns are n and c since it has been assumed that the cost components have been evaluated. Therefore, the proposed method of solution is to substitute values for n and c into the expression to obtain a value of the total cost for each pair of values (n,c) . By comparison of these values the sampling plan which results in the lowest overall cost can be chosen.

The final form of the cost model is then

$$\begin{aligned}
 E(C(n,c)) = & \bar{C}_m N \\
 & + C_S P'_S \\
 & + a\bar{C}_D P'_D \\
 & + (1-a)C_R P'_D \\
 & + (\bar{U} + a\bar{C}_D + (1-a)C_R) P_U \\
 & + \bar{C}_A + C_A n.
 \end{aligned}$$

The computer program as listed in Appendix 1 was developed for use in solving the above model. The values of P'_s , P'_D and P_U are evaluated depending on the input values of (n, c) . The total cost is calculated from these probabilities.

The total cost of a specific sampling plan for given costs, and overall process percent defective, is calculated by the program and the sampling plan and its total cost are printed. The total cost is then recalculated for a sampling plan with a smaller positive sample size but unvarying acceptance number. If the new total cost is larger than the old total cost, the program looks for a new sampling plan $(n$ and $c)$. Otherwise the sample size is again reduced and the procedure is repeated. If the sample size becomes non-positive or there is no data for a new sampling plan, the program terminates. Figure 2 is a flow chart of the computer program.

The program clearly obtains a local minimum for each acceptance number. It must be determined which sampling plan results in the global minimum by comparison of the local minima. It appears that this method of determining the optimum solution to the cost model is manageable since there is a finite number of sample sizes $(n = 0, 1, \dots, N)$ and acceptance numbers $(c = 0, 1, \dots, n)$ to be tried in the program's data.

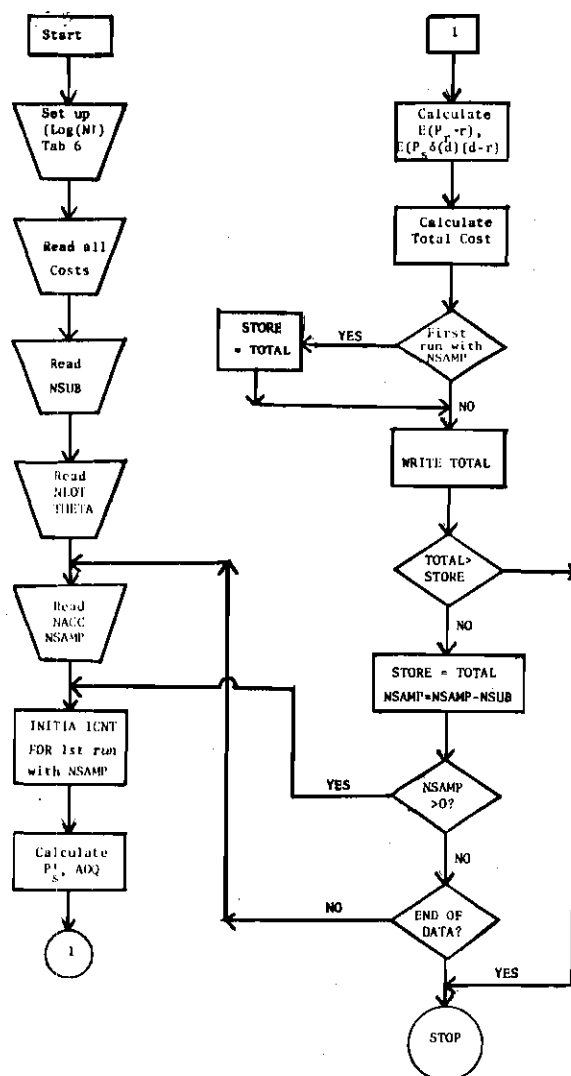


Figure 2. Flow Chart of the Computer Program

Sensitivity of the Model

The computer program in Appendix 1 was run with various sets of costs and process percents defective to obtain an idea of the model's sensitivity to these factors. It was necessary to "test" sensitivity by this method since there are numerous variables.

In testing the sensitivity of the model to the costs, the cost of screening a lot, C_s , the cost of replacing a defective item, \bar{C}_D , the cost of reworking a defective item, C_R , the cost of loss of customer goodwill, \bar{U} , and the per item cost of inspection, C_A , where the costs varied because they are the costs which can alter the optimal sampling plan. In general, the sensitivity analysis showed that only a fair estimate of these costs is necessary. The optimal sampling plan could rarely be changed by a slight fluctuation in the costs. The following table is evidence of this statement. Throughout Tables 1 and 2 $AOQ^* = .01$, $N = 500$, $\theta = .03$, and the components $\bar{C}_m N$ and \bar{C}_A , have been eliminated since they have no effect when solving for the optimal sampling plan. It should also be noted that the model yielded second choice sampling plans which are more conventional and involve more inspection but less than ten percent variance in the total relevant cost. Table 2 illustrates these second choice plans.

All but one of the optimal plans provides for an

Table 1. Optimal Plan Under Varying Costs

C_s	C_D	C_R	U	C_A	Optional Plan	Cost
.40	6.0	4.0	500.	.001	(35,0)	49.25
.40	6.0	4.0	250.	.001	(35,0)	49.25
.40	6.0	4.0	1000.	.001	(35,0)	49.25
.40	6.0	4.0	500.	.100	(35,0)	52.72
.40	6.0	4.0	500.	.010	(35,0)	49.57
.40	6.0	4.0	500.	.001	(35,0)	49.25
.40	5.0	4.0	500.	.001	(34,0)	44.46
.40	6.0	3.0	500.	.001	(34,0)	42.45
.30	6.0	4.0	500.	.001	(70,1)	50.23

Table 2. Second Choice Plans

C_s	C_D	C_R	U	C_A	Second Plan	Cost	Δ Cost
.40	6.0	4.0	500.	.001	(70,1)	50.59	1.34
.40	6.0	4.0	250.	.001	(70,1)	49.85	0.59
.40	6.0	4.0	1000.	.001	(75,1)	51.65	2.40
.40	6.0	4.0	500.	.100	(70,1)	57.52	4.80
.40	6.0	4.0	500.	.010	(70,1)	51.22	1.65
.40	6.0	4.0	500.	.001	(70,1)	50.59	1.34
.40	5.0	4.0	500.	.001	(72,1)	45.98	1.32
.40	6.0	3.0	500.	.001	(72,1)	43.90	1.45
.30	6.0	4.0	500.	.001	(40,0)	52.67	2.14

acceptance number of zero. The cost of screening a lot appears to be so low that the model's solution indicates that a plan with a poor producer's risk is appropriate.

The model was also tested to determine the sensitivity to the overall process percent defective. For the examples in Table 3, θ ran from .005 to .100 and $C_s = .40$, $\bar{C}_D = 6.0$, $C_R = 4.0$, $\bar{U} = 500.0$, $C_A = .001$, $AOQ^* = .01$, $N = 500$.

Table 3 shows that as the process percent defective increases, the cost of a lot also increases. The sample size varies only slightly until a high percent defective forces use of a strict sampling plan to avoid unsatisfactory-product costs. The jump in the sample size is caused by high unsatisfactory-product costs occurring earlier than with a smaller percent defective. These costs are zero until a minimum AOQ is violated and after violation they are large. For Theta greater than or equal to .080, Table 3 demonstrates this phenomenon. This result indicates that rectification may become unjustified when the process fraction defective is significantly greater than AOQ^* . In this case the process should be altered.

Validity of the Model

To show the validity of the model is at minimum, difficult. In the previous section sensitivity to costs and the overall process percent defective was discussed with the conclusion that the model is robust. In fact, the model is

Table 3. Optimal Plan Under Varying Theta

θ	Optimal Plan	Cost
.005	no sampling	--
.010	no sampling	--
.015	(24,0)	12.30
.020	(32,0)	24.69
.025	(34,0)	36.62
.030	(34,0)	48.48
.035	(34,0)	61.01
.040	(32,0)	73.47
.045	(32,0)	73.84
.050	(30,0)	75.21
.055	(54,1)	109.42
.060	(28,0)	120.65
.065	(28,0)	133.98
.070	(26,0)	145.25
.075	(26,0)	157.90
.080	(95,0)	192.36
.090	(95,0)	216.38
.100	(95,0)	240.37

so robust with respect to the costs that only vague estimates of them are sufficient for successful implementation of the model. This leads to questioning the advantages of the model. If the cost components influence the optimal sampling plan so slightly, perhaps there is little to be gained by use of the model.

The difficulties of the model must then be weighed against its advantages. The computer program used to solve the program does require large blocks of computer time. Once the model has been solved for the optimal sampling plan there is no need to use computer time again unless a cost component changes drastically.

Another disadvantage is that the cost of screening a lot, C_s , is determined on a per lot basis. Since the remainder lot is screened and not the entire lot, this cost should be developed on a per unit screened basis. C_s is assumed to be a function and the sample size could be used as one of its parameters, but in the example used in this thesis, this was not done.

The unsatisfactory cost expression is weighed by the extent of violation of the minimum average outgoing quality standard. In the case where the lots are sold to another manufacturer the expression accurately portrays the situation. That is, if he receives a lot within specifications he is satisfied.

In the case where the lots are sold to a retailer, the expression probably does not portray the situation. If an individual receives a defective item, he sees that his item is defective and does not consider that his defective may be one in thousands of items sold. He is clearly dissatisfied and his dissatisfaction is not reflected in the cost equation.

CHAPTER VI
CONCLUSIONS AND RECOMMENDATIONS
FOR FURTHER STUDY

The design of a single-sampling attribute inspection plan on the basis of economics is the purpose for developing the cost model presented in this thesis. The model can be easily adapted to individual problems and after all the cost components have been evaluated, solved with little difficulty.

A computer program is included and has been designed to find the probabilities associated with the cost components. The sampling plan which results in the lowest overall cost is found by using the trial and error method of solution.

It is necessary to know the distribution of the overall process percent defective. To obtain some knowledge of the sensitivity of the model (with respect to changes in cost), the binomial distribution was used for the distribution of the overall process defective along with arbitrary cost data.

The conclusions of the test runs of the computer program indicate that use of the model will guarantee a minimum cost sampling plan when the costs are "ballpark" estimates and the process fraction defective is accurate to $\pm .010$.

There is great potential in the field of quality control for the type of model discussed in this thesis, although more research is needed. Areas of research related to this model could be found within the following suggestions:

1. Investigate the cost components related to acceptance sampling. Specifically, the relationship of unsatisfactory costs to the optimum sampling plan should be studied as well as the relationship to other costs.
2. Determine how to more effectively use knowledge of the process percent defective.
3. Construct a set of tables or charts which will allow easy use of the model developed in this thesis.
4. Revise the program used in this thesis to calculate the optimal sampling plan so that it would use less CPU time.
5. Evaluate the sensitivity of the model on the basis of an experimental design model. This would provide for use of statistical methods for drawing conclusions and obtaining information on the effects of the model to changes in the parameters.

APPENDIX

APPENDIX I

The FORTRAN program included in this Appendix is appropriate for all situations where the lot size is less than 3001. For larger lot sizes one array must be redimensioned.

The variables which must be read into the program for each run are:

FACT:	An array containing Log (N!) where N is also the element number in the array
COSTS:	Cost of screening
COSTD:	Cost of replacing a defective
COSTR:	Cost of reworking a defective
U:	Average cost for the loss of customer good will
AOQ:	A minimum acceptable level of Average Outgoing Quality
COSTAB:	A per item sampled cost
O:	Fraction of defectives which must be replaced
NLOT:	Number of items in the lot
NSAMP:	Number of items in the sample
NSUB:	Amount by which NSAMP decreases
THETA:	Overall process fraction defective
NACC:	Acceptance number


```

DOUBLE PRECISION FACT(3000)
READ(5,11)(FACT(IK),IK=1,3000)
5001 WRITE(6,31)
31  FORMAT('1',5X,'INPUT COSTS')
    READ(5,10,END=5000) COSTS,COSTD,COSTR,U,A0Q,COSTAB
    WRITE(6,61) COSTS,COSTD,COSTR,U,A0Q,COSTAB
61  FORMAT(5X,7F10.5,5X,'INPUT THE NUMBER TO DECREASE SAMPLE BY,')
    READ(5,10) NSUB
    WRITE(6,62) NSUB
62  FORMAT(5X,I5)
    WRITE(6,40)
    O=.4
    READ(5,10) NLOT,THETA
    WRITE(6,70) NLOT,THETA
    WRITE(6,60)
    E1=FLOAT(NLOT)
500 READ(5,10,END=5001) NSAMP,NACC
51  ICNT=1
50  AOQC=0.0
    ICNT=ICNT+1
    N=NLOT-NSAMP
    PDM=0.0
    PD=0.0
    PS=0.0
    DO 100 ID=0,N
    K=MIN0(ID,NACC)
    DO 100 IR=0,K
    A=FACT(N)-(FACT(ID-IR)+FACT(N-ID+IR))
    IF ((ID-IR).EQ.0) A=0.0
    IF ((N-ID+IR).EQ.0) A=0.0
    B=FACT(NSAMP)-(FACT(IR)+FACT(NSAMP-IR))
    IF (IR.EQ.0) B=0.0
    C=ID*LOG(THETA)
    D=(NLOT-ID)*LOG(1.-THETA)
    IF (ID.EQ.0) GO TO 90
    E=FLOAT(ID)
    F=LOG(E)-LOG(E1)
    AOQC1=EXP(A+B+C+D+F)
    AOQC=AOQC+AOQC1
90  PS1=EXP(A+B+C+D)
100 PS=PS+PS1
    PS=1.-PS
    DO 200 ID1=0,N
    K1=MIN0(ID1,NSAMP)
    DO 200 IR1=0,K1
    G=FACT(N)-(FACT(ID1-IR1)+FACT(N-ID1+IR1))

```

```

H=FACT(NSAMP)-(FACT(IR1)+FACT(NSAMP-IR1))
IF ((ID1-IR1).EQ.0) G=0.0
IF ((N-ID1+IR1).EQ.0) G=0.0
IF ((NSAMP-IR1).EQ.0) H=0.0
IF (IR1.EQ.0) H=0.0
P=ID1*LOG(THETA)
Q=(NLOT-ID1)*LOG(1.-THETA)
IF ((ID1-IR1).EQ.0) GO TO 190
S=FLOAT(ID1-IR1)
T=LOG(S)
PDM1=EXP(T+G+H+P+Q)
PDM=PDM+PDM1
190 IF (IR1.EQ.0) GO TO 200
S1=FLOAT(IR1)
T1=LOG(S1)
PD1=EXP(G+H+P+Q+T1)
PD=PD+PD1
200 CONTINUE
CDPD=(PD+PS*PDM)*0
CRPD=(PD+PS*PDM)*(1.-0)
C NOTE THAT ALL THE COSTS CAN BE FUNCTIONS. ONE SUBPROGRAM IS
C REQUIRED FOR EACH FUNCTION.
X=COSTS*PS
Y=COSTD*CDPD
Z=COSTR*CRPD
V=(U+0*COSTD+(1.-0)*COSTR)*(AOQC-AOQ)*THETA*NLOT
IF (AOQC.LE.AOQ) V=0.0
W= COSTAB*NSAMP
TOTAL=X+Y+Z+V+W
IF (ICNT.EQ.2) STORE=TOTAL
WRITE(6,30) NSAMP,NACC,TOTAL
IF (TOTAL.GT.STORE) GO TO 500
STORE=TOTAL
NSAMP=NSAMP-NSUB
IF (NSAMP.GT.0 ) GO TO 50
NSAMP=NSAMP+NSUB-1
IF (NSAMP.GT.0) GO TO 50
GO TO 500
11 FORMAT(D8.7,5D12.7/499(6D12.7/))
10 FORMAT( )
40 FORMAT(25X,'TOTAL COST RESULTING FROM SAMPLING PLANS OF',
0'PARAMETERS(SAMPLE SIZE, ACCEPTANCE NUMBER),')
30 FORMAT(8X,I6,14X,I7,
0 14X,4(F15.7,1X) )
60 FORMAT(5X,'SAMPLE SIZE',5X,'ACCEPTANCE NUMBER'/)
70 FORMAT(56X,'THE LOT SIZE IS',I7/60X,'THETA IS',F5.4/)
5000 STOP
END

```

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